The Book Review Column[1]

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Please allow me to introduce myself: I, Fred Green, am on the faculty in the Math/CS department at Clark University. My primary research interests are in classical and quantum computational complexity, and you can learn a little more about me at http://mathcs.clarku.edu/~fgreen. As of this column I am taking over from the previous editor, Bill Gasarch. I thank Bill for (at least) three things:

1. Seventeen (17) long years of dedicated, invaluable and first rate service to the community as editor of this column. If this were a banquet or something I'd offer a toast. Even though it isn't, I hope as you read this you're on your feet with your glasses raised. We all owe him a debt of gratitude.
   Thank you Bill!

2. Offering my name as his successor. I am deeply honored.

3. Generosity above and beyond the call of duty in ensuring a smooth transition. This issue (and possibly the next) would have been devoid of reviews without his help!

Indeed, what finer tribute can we offer than to actually read the reviews he so enjoys writing. And so. . . [a drum roll please]. . . in celebration of his many years in this post, and so that you may rest assured that we have not heard the last of him on these pages, this column will consist entirely of *six* Gasarchian reviews (of *seven* books). I fully expect his contributions to continue, albeit not quite at the density represented in this issue. (We will return to our regularly scheduled programming next time.)

The focus and format of the column will, at least for the time being, remain substantially similar. The format will evolve, no doubt. For example, it might be interesting to have, on occasion, fewer but much longer and technical reviews (or, as appropriate, more but less detailed reviews). My hope is also that any changes will be driven, at least in part, by you, the readers and the reviewers. Please feel free to send suggestions to me, at the above e-mail. Ideas regarding the column as well as items to review are all very welcome. Speaking of items that need reviewing, please refer (as usual) to the coming pages, and let me know which title(s) interest you most.

There is something of a theme to this issue's reviews, modulo the fact that William Gasarch wrote them all. Each deals in some way with history, a particular computer scientist or mathematician, or that person's view of his field, and sometimes all three. They are (proceeding, very roughly, in historical order):

1. **The Cult of Pythagoras: Math and Myths**. This book, by Alberto A. Martinez, is much less about Pythagoras (if indeed he was anyone) than it is about the nature of mathematics from antiquity to the present day. Besides setting readers straight on some of the misinformation often propagated about Pythagoras, the book also examines the boundary between "invention" and "discovery."

2. **Infinitesimal: How a dangerous mathematical theory shaped the modern world**, by Amir Alexander. Dangerous. . . *really*? It seems that's what the church thought for centuries. It is fascinating to learn how this enormously powerful concept (invention? discovery? – see the "Cult of Pythagoras" review) prevailed in some places and not in others.

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3. **Martin Gardner in the Twenty-First Century**, edited by Michael Henle and Brian Hopkins, a book containing a number of articles by the famous Scientific American columnist and author, and quite a few serious mathematical papers mostly by other mathematicians, on a diverse array of subjects, inspired by his work.

4. **Algorithmic Barriers Falling: P=NP?**, and **The Essential Knuth**, two short books of interviews of Donald Knuth, both by Edgar Daylight, containing many insights, often surprising, about a founder of our field and his perspectives on it.

5. **Love and Math: The Heart of Hidden Reality**, mathematician Edward Frenkel's memoir and homage to his field, including high-level descriptions of quite a bit of substantial mathematics.

6. **Structure and Randomness: Pages from Year One of a Mathematical Blog**, by Terence Tao, based on his blog "What's New" (`https://terrytao.wordpress.com/`), and the most mathematically detailed of the books reviewed in this issue.

BOOKS THAT NEED REVIEWERS FOR THE SIGACT NEWS COLUMN

Algorithms

1. *ReCombinatorics: The algorithmics of ancestral recombination graphs and phylogenic networks* by Gusfield.
2. *Distributed Systems: An algorithmic approach (second edition)* by Ghosh.
3. *Tractability: Practical approach to Hard Problems* Edited by Bordeaux, Hamadi, Kohli.
4. *Recent progress in the Boolean Domain* Edited by Bernd Steinbach
5. *Distributed computing through combinatorial topology* by Herlihy, Kozlov, and Rajsbaum.

Programming Languages

1. *Selected Papers on Computer Languages* by Donald Knuth.

Miscellaneous Computer Science

1. *Introduction to reversible computing* by Perumalla.
2. *Algebraic Geometry Modeling in Information Theory* Edited by Edgar Moro.
3. *Digital Logic Design: A Rigorous Approach* by Even and Medina.
4. *Communication Networks: An Optimization, Control, and Stochastic Networks Perspective* by Srikant and Ying.
5. *CoCo: The colorful history of Tandy's Underdog Computer* by Boisy Pitre and Bill Loguidice.

Cryptography

1. *The Mathematics of Encryption: An Elementary Introduction,* by Margaret Cozzens and Steven J. Miller.

Miscellaneous Mathematics

1. *Asymptopia,* by Joel Spencer with Laura Florescu.
2. *Spectra of Graphs,* by Andries E. Brouwer and Willem H. Haemers.

Mathematics and History

1. *Professor Stewart's Casebook of Mathematical Mysteries* by Ian Stewart.
2. *The Golden Ratio and Fibonacci Numbers* by Richard Dunlap.
3. *Mathematics Galore! The first five years of the St. Marks Institute of Mathematics* by Tanton.
4. *Mathematics Everywhere* Edited by Aigner and Behrends.
5. *An Episodic History of Mathematics: Mathematical Culture Through Problem Solving* by Krantz.
6. *Proof Analysis: A Contribution to Hilbert's Last Problem* by Negri and Von Plato.

Review of[2]

The Cult of Pythagoras: Math and Myths
by Alberto A. Martinez
Publisher: University of Pittsburgh Press
$18.00 hardcopy, 288 pages, Year: 2013

Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

Excerpt from a popular Math history book from 3000 AD written by Milt Peel Creble.
Monty Hall was a great Mathematician in the late 1900's who worked in probability. He challenged some of the hidden assumptions in probability. He used various paradoxes to test his theories. One became quite popular and was called The Monty Hall Paradox. *Alas, mathematicians of his day counted his popular work against him. Hence he never received the Nobel Prize in Mathematics.*

What is wrong with this narrative?

- Monty Hall is not a mathematician.
- Monty Hall did not invent the Monty Hall Paradox. Steve Selvin did.
- The Monty Hall Paradox is not really a paradox. It was never used to test anything nor is it problematic.
- There was no Nobel Prize in mathematics until the year 2500.

Could this absurd and clearly false story ever become a well known story? Alas yes. The book under review shows the several oft-told stories about Pythagoras are as absurd. He also discusses other myths.

Another excerpt from a popular Math history book from 3000 AD written by Milt Peel Creble.
Monty Hall discovered the compact numbers which were used to resolve his paradox. At the time some old fashioned mathematicians claimed incorrectly that the compact numbers do not exist; however, much like the complex numbers, the quaternions, and the surreals, the compact numbers are now accepted by all mathematicians.

What is wrong with this narrative (aside from the issues raised in the last narrative)?

1. The Monty Hall Paradox does not need to be resolved.
2. There are no such thing as compact numbers. At least not yet.
3. The author of the book under review would say that the compact numbers were *invented* to resolve the paradox. To say they were *discovered* seems very very odd.

The author discusses whether math is invented or discovered. His own opinion is that much more of it is invented than we tell our students (or perhaps than we even realize). He also urges us to not hide these issues and historical debates from students.

[2]©2015 William Gasarch

2 Pythagoras

The information we have on Pythagoras is very slim. So-called historians in the past would use phrases like *as is well known Pythagoras provided the first rigorous proof of the Pythagorean Theorem* when this was not well known and there was no evidence for it. Similarly there is no evidence for the story that when one of his followers discovered a number that was not rational (the hypotenuse of a right triangle with both legs of length 1) that follower was (1) put to death or (2) accidentally drowned, which the Pythagoreans thought was a sign from God. (I had heard both versions.)

The author also speculates some on how these stories could have evolved. Alas, there is also scant evidence to go on here as well. My own opinion is that history is not just written by the winners, but is also written to make stories neat and clean and have a point, even if there is no such point in those stories. I give a non-math example of a story I heard in my youth:

Charles R. Drew was an African-American doctor who helped perfect blood transfusions. He died in 1950 when, after a car accident, the hospital he was at refused to give him a blood transfusion because he was black.

This story is not true. It was common in the 1950's for a hospital to only treat whites, and there was a notion (without any scientific foundation) that it was bad (morally? healthy-wise?) to give a black man white blood or vice-versa. Hence the story is great at conveying these points and to make us feel superior since we don't think that way anymore. But, alas, it's just not true.

Pythagoras permeates the entire book. Bertrand Russell criticizes Pythagoras for introducing Platonism (which he did not). Carl Boyer (a historian) comes close to claiming that Pythagoras invented or discovered (hard to tell which) infinitesimals (which he did not).

3 Gauss and Galois

One of the most well-told tales is that in first grade Gauss summed the numbers $\{1, 2, \ldots, 100\}$ quickly using a trick that is now well known. There are many versions of the story. While the story has very little foundation, at least (unlike the Pythagoras story) the point it's trying to make—that Gauss was exceptional at math at a young age— is true. Or so I've been told.

Galois did indeed die at a young age in a duel. Much else you've heard about him is speculation. And no, he did not write down all of his math the night before the duel.

4 Different Types of Numbers

There had been a myth that complex numbers confused Euler (I had not heard that one– but I did hear a myth that Euler was confused by infinite series.) This book debunks that and takes us on a tour of some nice mathematics as well. The issue at hand is how to define $\sqrt{a}$. Even for $a \geq 0$ this is not clear- is it the positive square root, or is it multivalued? No matter how you define it some standard rule of mathematics will fail.

This brings up the other point of this book: Math has had controversies. We should teach them! Math is more interesting and more accurate when we tell students that math did not arise in the pristine form that it ends up in in textbooks. The story of complex numbers and quaternions are examples that are discussed.

Are questions of number systems all settled now? Surprisingly no! What is $1/0$? Infinity? Undefined? Or as some computers say NaN (Not a Number). There are still competing systems. Computers have made this more relevant since $1/0$ may come up naturally and not as a mistake. Even in pure math it may be relevant since projective geometry has a point at infinity.

The author also writes of the very different views of infinitesimals. The most amusing one is that Leibniz thought they were a useful fiction, but that others agreed but didn't want this known. The author writes only of the controversy about infinitesimals taking place in the north (England and Germany) where it was mostly a mathematical debate. The book *Infinitesimal* (the next review in this column) by Amir Alexander tells the story of the controversy over infinitesimals in Italy where it was mixed in with the Catholic Church who were against infinitesimals and banned their use.

The author invents his own number system where $(-1) \times (-1) = (-1)$. His claim is not that we should use it, but that we should not automatically reject it. Note that complex numbers, quaternions, and transfinite numbers were at one time rejected. The author does mention surreals; however, they seem to have never caused controversy.

The author ponders if π really would be the best number to beam over to an alien civilization. He does not mention the τ-movement, that a better constant for math would be τ which is the ratio of the circumference to the diameter, actually 2π. Some believe we should use τ since 2π occurs in formulas more than π does[3].

5 Geometry

There is an entire chapter on non-euclidean Geometry. The history is interesting. The point is that Geometry was once thought of as being an absolute— so theorems in it were thought to be discovered; however, now that we know there are many geometries, it might be better to think that geometry is invented.

6 Summary

The author has a strong point of view on the nature of mathematics and presents it with authority. Even if you disagree with him, in fact especially if you disagree with him, this book is worth reading.

His point of view is that some parts of math are discovered (that is, exist independent of our knowing about them) but many are invented. While I don't quite know where he draws the line, it's far earlier than I do— he seems to think that $(-1)(-1) = 1$ is invented.

I used to draw the line at large cardinals. My wife thinks the Banach-Tarski paradox proves math is broken, so she would say the Axiom of Choice was surely invented and is false.

One question that is not quite addressed: why do some systems win out? Is it because they are more useful? Is it politics? This would be a study unto itself.

I close by saying when I first had thoughts like the author. I noticed that $5^{1/2} = \sqrt{5}$ and wondered if there was some sense in which we are multiplying 5 by itself 1/2-a-time. There is not! We define $5^{1/2} = \sqrt{5}$ so that the rule $a^b \times a^c = a^{b+c}$ is true. In my humble opinion *there is no other reason to define it that way*. I have no problem defining it that way, but we should admit that it's a convention we invented so that an old rule still holds, and not a discovered law of nature.

Milt Peel Creble is an anagram of a real person's name. I discovered (invented?) this anagram. I challenge the reader to determine who it is an anagram of.

[3]When I blogged about this Terry Tao wondered if $2\pi i$ might be even better. He went on to win the Fields Medal and the Breakthrough award, though not for that observation.

Review of
Infinitesimal
How a dangerous mathematical theory shaped the modern world[4]
by Amir Alexander
Publisher: Scientific American/FSG
$20.00 hardcopy, $11.00 Kindle

Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

We have probably all heard the following:

- In the late 1600's Newton, Leibniz, and others based calculus on infinitesimals which were not rigorous, but calculus worked.
- In the early 1800's Cauchy, Dedekind, and others put calculus on a rigorous basis with the formal definition of the reals and of limits.
- Abraham Robinson made infinitesimals rigorous in the 1960's using techniques of model theory that were not avaiable to the mathematicians in either the 1600's or the 1800's.

Hence infinitesimals hardly seem like *a dangerous mathematical theory*. This book does not really talk about those historical episodes. Instead it talks about a slightly earlier time when some people thought reasoning using infinitesimals was *dangerous*.

How could a non-rigorous method of proof that seemed to produce the right answers be dangerous? One might wonder if such techniques can be made rigorous, or one can wonder if in some sense they already are rigorous, or one can also wonder if the results proven really are true. But this would seem to be a discussion among mathematicians, and hardly dangerous.

That last paragraph was written by a 21st century person (me). To truly understand how a mathematical theory can be considered dangerous in the mid 1500's we need to know the history and context. This book provides that as it tells two tales:

- How infinitesimals lost the battle of ideas in Italy.
- How infinitesimals won the battle of ideas in England.

2 Summary

2.1 Part I: The War Against Disorder: The Jesuits Against the Infinitely Small

Before the reformation most (perhaps all) Christians were Catholic. The reformation (1517) cast doubt on long held religious beliefs. How could the Catholic Church regain its position? The Jesuits were formed (1540) to strengthen the Catholic position by teaching and argument. They founded many colleges and succeeded to some extend. Their colleges would look very strange from a modern viewpoint: they *discouraged*

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original thought since there was no room for debate on the truth which was absolute. Expositions of older work (e.g., Euclid) were fine but original research was frowned upon.

They sought to put theology on as rigorous a basis as mathematics. Hence any mathematics that was not rigorous was considered dangerous. Several Italian mathematicians (including Galileo) used infinitesimals in a variety of ways and their work was condemned. The reasons for condemning it were a strange mix of theology and mathematics- topics that are separate today but were more connected back then. In the end the theory died in Italy.

2.2 Part II: Leviathan and the Infinitesimal

The English Civil war devastated England and raised many questions about what is the best form of government. David Hobbes thought he had the answer. In his book *Leviathan* he argues that the best government is one which has an absolute ruler and absolutely no dissent. That way there could be no rebellions or civil wars. He also thought that this could be proven *mathematically*. Moreover he thought that in order to do this, geometry must be able to resolve every question. Hence he worked on the problem of using a straightedge and compass to square the circle and had a construction that he was convinced worked (it did not, and in fact the problem is impossible). As a 21st centuery person I think that either squaring the circle or proving it could not be done would settle the question; however, I doubt he would see it that way.

He despised infinitesimals since they did not fit how he thought proofs should go. Unlike the Jesuits, his arguments against infinitesimals were math-based. However, his arguments did not carry the day. Partially because he also defended his incorrect construction of squaring the circle and partially because John Wallis, a far better mathematrician, was using infinitesimals with great success. In the end infinitesimals were accepted in England.

2.3 What to Make of All This?

In the mid 1500's if you were to ask which of Italy or England would go on to make great contributions to Mathematics and Science, the answer would clearly be Italy. What went wrong? The author claims that Italy's rejection of infinitesimals, and England's embracing of them, was the cause. This seems a bit simplistic; however, there is clearly some truth in it. It may also be that the kind of society that lets religion (or other non-science factors) dictate science will fall behind those that do not.

2.4 What I Hope is in a Sequel

In the 1600's the Church had an opinion about techniques of proof in mathematics. In the late 1800's Cantor inquired of the Catholic church if they were okay with his theory of cardinals. They were. I wonder if they cared. I'd be curious what the Catholic Church would have thought of the Banach-Tarski paradox had it been discovered in the 1600's.

Doren Zeilberger (whose blog entry about this book inspired me to review it) believes that experimental mathematics can and should be the basis of proofs. What does the Catholic Church think of this? We cannot imagine they have an opinion! In short, in 2014 the Church does not have an opinion about techniques of proof in mathematics.

By the intermediate value theorem there must exist a year $1600 \leq x \leq 2014$ such that in year x the Church cared but in year $x + 1$ they did not (I know that this is simplistic). I would want to see a sequel that describes when the Catholic Church stopped caring about these things.

3 Opinion

This is a history book with some math in it (of interest!). Very little math background is needed for the book; however, one has to care about the issues involved.

The book tells a fascinating story of an age when some people thought that (1) mathematical proofs can apply to religion and politics, and (2) once this is done their opinion will be proven correct. These points of view look absurd today; however, we must keep in mind that we have the benefit of their failures to learn from.

Review of[5]

Martin Gardner in the Twenty-First Century:
Edited by Michael Henle and Brian Hopkins
Publisher: MAA
$40.00 Softcover, 300 pages, Year: 2013

Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

As I am sure the readers of this review know, Martin Gardner had a column in Scientific American on mathematics. Recreational? Deep? Fun? Serious? (these need not be opposites) Part of his genius is that he blurred that line. The book under review is proof of that blurring: it contains articles in serious math journals that were inspired by fun things he wrote in his column.

2 Content

There are 41 articles, 8 by Gardner himself, and the rest by a variety of mathematicians. They are in 7 categories: Geometry, Number theory and Graph theory, Flexagons and Catalan Numbers, Making things Fit, Further Puzzles and Games, Cards and Probability, and Other Aspects of Martin Gardner. Notice that this is an odd way to divide mathematics (e.g., *Number theory* and *Graph Theory* in the same category, and that category of equal importance to *Flexagons and Catalan Numbers*); however, this is what is so wonderful about this book— it's a collection of random math of interest and hence need not fit into any preconceived notions.

I describe an article from each section, except that I take two from the section *Number Theory and Graph Theory*.

Geometry: *Prince Rupert's Rectangles.* Let C be a $1 \times 1 \times 1$ cube. Can you pass another $1 \times 1 \times 1$ cube through C? You can. Note that this is the same as asking can you slice it and have the slice contain a unit square. The answer is YES and this is a much discussed problem. This chapter generalizes the problem in two ways: higher dimensions, and rectangles (actually the analog of rectangles in higher dimensions).

Number Theory and Graph Theory *Squaring, Cubing, and Cube Rooting.* Can you compute 455×782 in your head? One trick is to rewrite it as

$$(500 - 45)(800 - 18) = 500 * 800 - 45 * 800 - 18 * 500 + 45 * 18$$

the first product is easy. The rest can be made easy by similar tricks. This article discusses tricks for doing squares, cubes, and even cube roots in your head.

Number Theory and Graph Theory *The Map-Coloring Game.* It is well know that every planar graph is 4-colorable. So if Alice wants to color a planar graph by herself she can. But what if bumbling Bob wants to "Help". Alice colors a node, Bob colors a node, etc. Bob is not allowed to intentionally create an invalid color unless he is forced to. Alice does not want to, as we will see. If the graph ends up being colored then

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Alice wins. If the graph does not, then Bob wins. It is known that there are planar graphs such that Alice cannot win just using 4 colors. This article shows that Alice can always win with 18 colors. This article also gives a nice history of prior results.

Flexagons and Catalan Numbers *Convergence of a Catalan Series.* Let $C_n = \frac{\binom{2n}{n}}{n+1}$. It is easy to show that $\sum_{n=0}^{\infty} \frac{1}{C_n}$ converges. What does it converge to? This chapter answers that question and others. It uses differential equations and generating functions. While I would not call it recreational, I would call it fun!

Making Things Fit *Squaring the Plane.* The following result was proven in 1958 and popularized in a Martin Gardner column (which I found exciting when I first read it in a book that appeared in the 1970s). *There is a square that can be tiled by smaller squares, all of different sizes.* Later came the following result in the same spirit: *There is a tiling of the entire plane using squares with Fib-numbered sides*. The question arises: Is there a tiling of the plane using tiles of side 1,2,3,. . ., and each on exactly once. YES- read the article to find out how.

Further Puzzles and Games *RATWYT.* Wythoff NIM is the following well known (popularized by Martin Gardner) NIM-game: there are two piles of stones, on each move a player either removes as many as he likes from one pile or the same number from both, and as usual if a player can't move then he loses. This chapter defines a natural variant where the number of stones is rational. For each ordered pair of rationals (r_1, r_2) one can easily find an ordered pair of naturals (a, b) such that the game on (r_1, r_2) is equivalent to the usual natural number version of (a, b).

Cards and Probability. *The Secretary's Problem from the Applicants Point of View*. The following is well known: If there will be n applicants for a job (n is large) , and the boss can only compare one to the prior ones, then the strategy that maximizes the boss's probability of getting the best person is to ignore the first n/e of them and then pick the first one that is better than all prior ones (if none exist then the boss has to take the last applicant). This yields a probability of the boss hiring the best applicant of $1/e$. But what if you are a job applicant and you know the boss is using this strategy? What if you know there will be n applicants (including yourself) and you can choose *when* to interview (that is, be the first or the 8th or . . .). This chapter considers this problem both in the case that the applicant knows his rank, and that the applicant does not know his rank.

Other Aspects of Martin Gardner[6] *The Golden Ratio: A Contrary viewpoint.* Many articles on recreational math (though I doubt any by Gardner) have extolled the Golden Ratio: It's the most pleasing rectangles! It occurs in nature many times! Hogwash. These papers are mostly bogus and this article shows it.

Looking over these chapters and others I am amazed about how many problems that are now well known were popularized by Martin Gardner. For that, and much more, we all owe him a debt of gratitude.

3 Opinion

Let n be the number of chapters in this book. For all people p who like math there exists at least $2n/e$ chapters that they will enjoy and another $O(1)$ that they will get something out of.

[6] I think this should have just been titled *Misc*.

Joint Review of
Algorithmic Barriers Falling: P=NP?
by Donald E. Knuth and Edgar G. Daylight
Publisher: Lonely Scholar
$20.00 Paperback, 100 pages, 2014

and

The Essential Knuth
by Donald E. Knuth and Edgar G. Daylight
Publisher: Lonely Scholar
$15.00 Paperback, 90 pages, 2013
Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

Both of these books are Edgar Daylight interviewing Donald Knuth. They talk on many topics including computer science, mathematics, and the history of science. Given Donald Knuth's place in our history his perspective is worth listening to. *Algorithmic Barriers Falling: P=NP?* (henceforth ALG) is more about algorithms and theory, *The Essential Knuth* (henceforth KNU) is more about Knuth.

While Knuth is making his points he seems to *not* be criticizing others. I'm not saying *he is careful to not criticize others*, I think being nice just comes naturally to him. As an example, when discussing pointer machines and RAM's he says the following:

RAM's and pointer machines are polynomially equivalent. They differ only when we make finer distinctions, like between linear time and $n\alpha(n)$ *(*α *is the inverse of Ackerman's function). The pointer model hasn't become more popular than the RAM model, because complexity theorists are happiest with a model that makes it easiest to prove theorems.*

Those guys have a right to study polynomial fuzzy models, because those models identify fundamental aspects of computation. But such models aren't especially relevant to my own work as a programmer. I treat them with respect but I don't spend too much time with them when they're not going to help me with the practical problems.

2 Very Short Summary

Since the books are short, the review will also be short. I'll just give a list of points that struck me. If you read the books then you may generate a different list.

The following points from ALG struck me:

1. Math papers shouldn't hide how they got to their results.
2. Notation actually drives research. By not using $o(n)$ Knuth forced himself to obtain sharper results.
3. Much asymptotic work has no real application to computing.
4. In the 1960's compiler research represented about 1/3 of computer science. I doubt this is meant as a precise estimate; but suffice to say that it was a large part.

5. It's important to know the history of the field since observing how others created new results may help you to create new results.

6. The history of the Knuth part of the Knuth-Morris-Pratt algorithm: Cook had shown that any set of strings recognized by a 2-way PDA has a linear-time algorithm on a RAM. Knuth went through the construction for the case of pattern matching and came up with a usable algorithm. This surprised him since he didn't think automata theory would ever lead to a simple algorithm that he couldn't come up with using just his programmer's intuition.

7. Right after Cook's paper on (what is now called) the Cook-Levin Theorem there was *optimism*— all we need to do is show that SAT is in P and we'll have many other problems in P! This lasted for a few months.

8. LaTeX: while working on it he became an expert on typefaces and fonts. Each letter is somewhat complicated and involves 65 parameters.

9. Knuth thinks that $P = NP$ (yes, that $P = NP$) but that there will still be some sort of distinction within P. Problems that are NP-complete will still be hard, because we won't know explicit polytime algorithms for them — we'll only know that such algorithms exist.

The following points from KNU struck me:

1. Knuth is really a programmer at heart. He may need some esoteric piece of math for an analysis, but his real goal is faster or better programs. It is common in theory (in science? in life?) to use a simple model as a starting point for what you really want to study, but then mistake the model for reality. Knuth never fell into this trap.

2. Knuth was doing statistical analysis of sports (Basketball) way before the moneyball revolution.

3. Knuth wrote a tic-tac-toe program in the early days of computing when it required using symmetries to save space.

4. Dijkstra coined the term *the pleasantness problem* to mean the gap between what we specify (perhaps formally) what we want a program to do and what it really does.

5. Structured programming was a very big breakthrough; however, gotos are sometimes useful.

6. Knuth is bothered by the trend in History of Science to dumb down the science. If these two books were an attempt to counter that trend then they have succeeded. (Actually, even if that was not the intent then they have still succeeded.)

3 Opinion

These are wonderful books that give great insights from THE founder of algorithmic analysis. What is most remarkable is that he is truly a computer scientist — he will learn and come up with hard mathematics, but he never loses sight of the original goal: faster real world algorithms for real world problems.

Review of
Love and Math: The Heart of Hidden Reality[7] [8]
by Edward Frenkel
2013, Basic Books, $27.99, 292 pages
Review by William Gasarch gasarch@cs.umd.edu

1 Introduction

Mathematics has been around for thousands of years and hence has gotten the chance to become very complicated. Fields of mathematics often, over time, become connected in surprising ways. Edward Frenkel uses the analogy of different continents and bridges that are built between them. This is very apt since (1) building a bridge between (say) America and Europe would be very difficult, and (2) much of the math he talks about is very difficult.

This book is partially Edward Frankel's autobiography: how he got involved with mathematics, the problems he faced as a Jew in the old USSR, and his love of the subject. In order to convey this he also explains a great deal of mathematics. The mathematics is sophisticated (I will briefly present some later), yet he manages to give a sense of it. In particular he conveys that this material is interesting and important. While I was reading the book, I was also working on a fun but frankly non-important problem in mathematics. The contrast was striking!

2 Summary

The book– which is written in the first person– begins by describing some elementary particle physics and group theory and explaining how they relate. After this comes the author's own tale. Frenkel was born in 1968 in what was then the Soviet Union, and he initially wanted to study Physics. His mentor, Evgeny Evgenievich Petrov converted him to Mathematics, the subject in which he earned his PhD, but since his work often related to physics it is not clear he really did switch. At such high levels it is hard (and not productive) to distinguish the two.

As a young man Frenkel was a brilliant mathematician, and if the Soviet Union had not practised a form of institutionalized antisemitism, he would have passed his exams and got into Moscow University. But under the circumstances such sucess was impossible no matter how well he performed. The examiners kept making the questions harder and harder, both in terms of intellectual merit (which he as still able to solve) and in terms of stupid pedantics. As an example, he gave the definition of a circle as 'the set of points equidistant from a point' which was deemed wrong since it should be 'the set of all points equidistant from a point'. Another way for Jews to be barred from school was to give them harder problems to solve. Often they had an easy solution that was hard to find, thus giving the appearence of fairness. For more on this see *Jewish Problems* by Khovanova and Radul, here: http://arxiv.org/abs/1110.1556.

Frenkel's experience was far from unique, and in this book he describes the ways in which Jewish Mathematicians, Physicists, and other scientists dealt with this system. Some of them met quasi-secretly and still managed to get much done. It is tempting to wonder if this oppression got their creative juices

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[8]This review originally appeared in the January 2014 issue of *Physics World*, under the title "Mathematics and Prejudice." Reprinted by permission. The full text is at http://physicsworld.com/cws/article/indepth/2014/jan/23/mathematics-and-prejudice.

flowing; however, this is a fallacy. We only read about those who managed to do well. I am sure that many brilliant students were blocked from making contributions. Their biographies are not written.

I am curious about what caused this antisemitism and what its effect (surely negative) was on the Soviet Union. The book does not go into this since it is his story and does not pretend to be a sociology or history book. However, it is good to have his story documented.

At the heart of the book is the Langlands program which is an attempt to build those *bridges* between different mathematical *continents*. Here is an example. Let $f(x, y)$ be a cubic polynomial with integer coefficients. If p is a prime we can ask how many $(a, b) \in Z_p \times Z_p$ such that $f(a, b) = 0$. we denote this n_p. We can then form an infinite polynomial $g(x) = \sum_{p \text{ prime}} n_p x^p$. This infinite polynomial is then associated to a group G of symmetries in the complex plane called a modular form. This correspondence is 1-1 and preserves some properties. That is, every cubic polynomial maps to a modular form and every modular form maps to a cubic equation. This is an important connection.

What the Langlands program does is essentially take the notion of *polynomial* and generalize it, and take the notion of *modular form* and generalize that. The program then makes conjectures about how these very abstract objects relate. Frenkel illustrates this connection-building process with several nice examples until, on page 222, he has a chart that connects Number Theory, Riemann Surfaces (Geometry), and Quantum Physics. Quantum Physics? How did that get in there? Through Gauge Theory– a complicated notion that Frenkel (wisely) does not try to explain. However, having read this book I now want to find out what it is.

Frenkel claims that frequently a branch of math that was thought of as pure abstraction ends up being applied to practical problems. I am often skeptical of such claims since the (perhaps forgotten) origin of many mathematics problems is, in turn, some real-world application. However, the examples given here seem legitimate since Number Theory really has no apparent connection to Quantum Physics yet the links are there. One is left with the impression that Frenkel and others in the book (including Ed Witten, the only Physicist to win a Field's Medal) are serious brilliant people who are doing serious brilliant work.

3 Opinion

You do not need to know much math to read this book, but you need to like it. Depending on your level you will get lost at some point (I got lost on the definition of a "sheaf"). However, this is not a book to read to learn math. Its a book to read to be inspired to learn math.

Review of[9]
Structure and Randomness:
Pages from Year One of a Mathematical Blog
by Terence Tao
Publisher: AMS
$34.00 Softcover, 300 pages, Year: 2008

Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

In the movie *Oh God! Book II*, God (played by George Burns) says:

Mathematics, that was a mistake. I should have made the whole thing a little easier.

While I am not one to argue with God, George Burns, or George Burns playing God, I do not think Mathematics was a mistake. But I do wish it was easier. Or perhaps it's easy enough to make so much progress in that it becomes hard.

Terence Tao has a math blog that I try to read but find difficult. Often the mathematics itself is beyond me, but other times I have a sense that I really could understand it if I just gave it a bit more time. How to get that time? I can't really explain it, but having the blog in book form really makes a difference for me. I made the same comment when reviewing both books based on the blog *Gödel's Lost Letter* and will likely make the same comment if I review Scott Aaronson's upcoming blog-book. And I don't think it's my inner-Luddite talking, as many non-Luddites I've spoken to agree with me.

Making the blog entries into a book removes one obstacle. Now the question arises, is the book worth reading? The short answer is yes. The long answer is that there are several types of chapters– some I could read, some I really couldn't, and some are in between. I review the book by giving some examples of each type.

2 I Understood The Entire Chapter!

The chapter *Soft Analysis, Hard Analysis, and the Finite Convergence Principle* discusses the difference between Soft Analysis, which seeks theorems about infinite objects, and Hard Analysis which seeks theorems about finite objects and (the hard part) concrete bounds. His point is that these two are not that far apart and can help each other. He then gives a great example: The infinite cvg theorem from soft analysis. The theorem is:

Every bounded monotone sequence of reals converges.

He then discusses what the *finite cvg theorem* is, going through several candidates. The final theorem deserves to be the analog of the infinite convergence theorem since *the infinite cvg theorem and the finite cvg theorem are equivalent!* and he proves it. I understood this chapter so well that I gave a talk about it in seminar.

The chapter *The Crossing Number Inequality* presents the crossing number inequality and two applications, all with proofs.

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Let G be a graph, v be the number of vertices, e be the number of edges, and c be the crossing number (the least number c such that the graph can be drawn in the plane with c crossings). The Crossing Number Inequality states that if $e \geq 4v$ then $c = \Omega(e^3/v^2)$. (The constant obtained by Tao is $\frac{1}{64}$ though better is known with a stronger condition on e and v.)

Let P be a finite set of points in the plane. Let L be a finite set of lines in the plane. What is the maximum number of incidences of points in P on lines in L? The Szemerdi-Trotter theorem proved that it is bounded by $O(L^{2/3}P^{2/3} + L + P)$. Tao shows how this can be derived from the Crossing Number Inequality, which he credits to Szekely.

Given A, a set of reals, one can look at $A + A = \{x + y \mid x, y \in A\}$ and $AA = \{xy \mid x \in A, y \in A\}$. Must it be the case that one of these sets is large? Using the Szemerdi-Trotter theorem one can show that either $|A + A|$ or $|AA|$ is of size $\geq |A|^{1+0.25}$.

This is an excellent exposition of some math that is fairly easy and interesting. It also shows Tao's range: he can talk about hard math, easy math, pure math, applied math, and everything in between.

My favorite chapter was *Hilbert's Nullstellensatz* (henceforth HN). He begins with what is a common dilemma: the proof he has seen of HN is a bit too abstract (too abstract for Terence Tao-Yikes!), and not computational. So he came up with a proof that is more computational and less abstract. Is it new? This is the great thing about blogs— **I don't care!** Tao thinks it might be an old proof presented differently, but none of this is important. Whats important is that there is a proof of HN which I can and will present in seminar!

What is the HN? I state what he calls the weak HN:

Let F be a fixed algebraically closed field. Let $d \geq 1$. Let $P_1, \ldots, P_m \in F[x_1, \ldots, x_d]$. Then either

1. There exists $\vec{a} \in F^d$ such that $P_1(\vec{a}) = \cdots = P_m(\vec{a}) = 0$, or
2. There exists polynomials $Q_1, \ldots, Q_m \in F[x_1, \ldots, x_d]$ such that $P_1Q_1 + \cdots P_mQ_m = 1$.

3 I Understood Something Interesting from the Chapter!

Most chapters in the book are in this category: I got something out of it but it really was a shade (or several) over my head. Many of them are in the category of *Now I know that that piece of hard math relates to that other piece of hard math.*

The chapter *Ultrafilters, non-standard analysis, and ϵ-management* interested me since I've heard about *applications of logic to "real math"* and heard debates about the issue: are there any (clearly yes), are there many (not clear), and will the algebraic geometer of the future need to know model theory (most algebraic geometers hope the answer is no). Tao claims that using non-standard analysis and ultrafilters can clean up some proofs and he gives some examples.

The chapter *Ratner's Theorem* uses Ratner's theorem, which is about topological spaces and closures, to prove a result in number theory that any undergraduate can understand. I learned the following:

1. If Q is a positive definite quadratic form (which may have irrational coefficients) then $Q(Z^d)$ is a discrete set of positive reals.
2. If Q is a positive definite quadratic form with integer coefficients representing all positive naturals ≤ 290 then it represents all positive naturals.
3. If Q is not positive definite then can $Q(Z^d)$ be dense?

(a) If there are just two variables then no: Take $Q(x,y) = x^2 - \phi^2 y^2 = (x - \phi y)(x + \phi y)$ where ϕ is the golden ratio. There is an interval around 0 where there is no element of $\{Q(x,y) \mid x, y \in Z\}$.

(b) There is a difficult proof by Margulis that for all $Q(x,y,z)$ that are not positive definite and have irrational coefficients, $Q(Z^3)$ is dense.

(c) This result can also be obtained from Ratner's theorem easily.

4 What Else is in the Book?

There are chapters on physics, applied math, open problems. There are expositions of fields (Structure and Randomness, Arithmetic Combinatorics). Lets just say there is a lot in this book.

5 Opinion

Who should read this book? You have to already like mathematics and know some. An excellent undergraduate math major could get something out of some of the chapters. She may also be inspired to learn more. I can imagine any chapter becoming a reading-course or even a research project. The more math you know the more of it you can understand. But be warned- the sheer breadth of knowledge in this book will render some fraction of it not really understandable.

Terence Tao is an excellent writer. It's impressive that he can run a blog that is both well written and has rather hard math in it. The more you put into reading his blog (perhaps in book form), the more you'll get out of it.